

## SYNTHESIS OF SECOND-ORDER INVARIANT CONTROL SYSTEMS WITH LARGE GAIN RATIO

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**Abstract-** In the article, the question of whether increasing the amplification factor to a large extent does not adversely affect the stability of the system is considered. Increasing the gain coefficient of the regulator and identifier to the required value ensured its high speed and accurate tracking of the reference trajectory without disturbing the stability of the system. The comparative analyze of the proposed methods and algorithms were evaluated by modeling and studying test problems in MATLAB/Simulink.

**Keywords:** Invariant control Systems, Until Sufficient Estimation of the Perturbation, Lyapunov Function, Quadratic Function, Observer.

### 1. INTRODUCTION

Since the static error consists of two summaries, the controller must be selected in such a way that the statism (that is, the static error is equal to zero) is performed simultaneously, taking into account both the task and the excitation effect. As the value of the amplification factor increases, the oscillation increases and the stability reserves decrease. The article is devoted to the synthesis and research of control systems with a large amplification factor. A new solution of this type of systems is provided based on the Lyapunov function method. When using the Lyapunov function method, the control strategy is based on the condition that the time derivative of the Lyapunov function is less than zero.

The basis of the article is a new synthesis method based on the Lyapunov function of control systems with a large amplification factor. Since the equation of the system does not include the parameters of the object and external influences, the motion of the system is widely fully invariant. However, since the required quality indicators and stability reserves are limited in applied issues, it is possible to provide these indicators at a finite value of the amplification factor. Based on analytical and computer studies, it is possible to determine the shortcomings of existing management systems and algorithms. In the indicated direction, a single-loop structure of the control system was proposed, which allows to increase the amplification factor excessively without breaking the stability, which is the primary issue.

From simple and effective robust approaches that do not use adaptation tools, systems with a high amplification factor can be shown. The main scientific innovation in this direction is the new solution of the problem based on the Lyapunov function method. These systems are referred to as "threshold control systems" with a large amplification factor [2, 3].

In the considered approach, the basis of the synthesis is that the static error depends on the amplification factor of the open system. The solution of this problem, which seems simple at first glance, faces a fundamental difficulty - increasing the amplification factor leads to a violation of the stability of the closed system. Existing studies in this direction are focused on the solution of the indicated problem. The main issue is the creation of a structure that allows the infinite increase of the amplification factor without breaking the stability. The issue is brought to the structural synthesis, which currently has no solution. Heuristic approaches prevail.

The fundamental difference of the considered method is that the nominal model of the uncertain object is not used during the synthesis: for example, as in the identification algorithms of uncertainties. This feature is a very important advantage. The decision of transitional processes in inversely connected systems takes place at the price of infinity of time [1, 3]. The quality of the automatic control system in the steady state is characterized by the static error, which is the threshold value of the dynamic error:

$$\Delta_s = \varepsilon(\infty) = \lim_{t \rightarrow \infty} \varepsilon(t)$$

According to the principle of superposition, the static error in linear systems is equal to the sum of the static errors  $\Delta_i$  caused by all inputs:  $\Delta_s = \Delta_1 + \Delta_2 + \dots$

Let's look at the one-dimensional automatic control system shown in Figure 1.

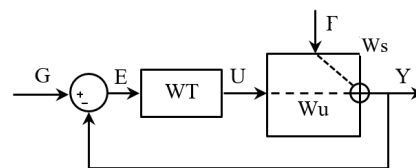


Figure 1. One-dimensional automatic control system

Since there are two inputs in a one-dimensional automatic control system considering input and output

$$\Delta_s = \Delta_s^g + \Delta_s^f, \Delta_s^g = \lim_{t \rightarrow \infty} \varepsilon_g(t) \quad (1)$$

$$\Delta_s^f = \lim_{t \rightarrow \infty} \varepsilon_f(t)$$

If the corresponding error descriptions are known, then by the limit theorem:

$$\Delta_s = \Delta_s^g + \Delta_s^f = \lim_{s \rightarrow 0} sE_g(s) + \lim_{s \rightarrow 0} sE_f(s)$$

$$E_g(s) = W_g^\varepsilon G(s), E_f(s) = W_f^\varepsilon F(s)$$

$$W_g^\varepsilon = \frac{1}{1+W_A}, W_f^\varepsilon = \frac{W_f}{1+W_A} \quad (2)$$

As mentioned, the static error is calculated using the following expression:

$$\Delta_s = \Delta_s^g + \Delta_s^f = \lim_{s \rightarrow 0} sW_g^\varepsilon G(s) + \lim_{s \rightarrow 0} sW_f^\varepsilon F(s)$$

Suppose the transfer function of the open system is as follows [5].

$$W_g^\varepsilon(s) = \frac{1}{1+W_A(s)} \quad (3)$$

$$W_f^\varepsilon(s) = -\frac{W_f(s)}{1+W_A(s)}$$

where,  $W_A(s)$  is the transfer function of the open system and is equal to the product of the transfer functions of the loops (regulator and object) in the direct circuit:

$$W_A(s) = W_T(s)W_u(s)$$

The amplification coefficient of the open system is equal to the product of the amplification coefficients of individual tubes:

$$k_A = k_T k_u$$

They usually change the gain of the controller to affect the dynamics and statics of the system [11, 12]. In this case

$$W_g^\varepsilon(s) = \frac{1}{1+k_T W_M W_u}, W_f^\varepsilon(s) = \frac{W_f}{1+k_T W_M W_u}$$

$$\Delta_s^f = \lim_{s \rightarrow 0} \left( \frac{W_f(s)}{1+k_T W_M W_u} \right) \times \frac{1}{s} = \frac{W_f(0)}{1+k_T W_u(0)} = \frac{k_f}{1+k_T k_u} \quad (6)$$

Suppose that,

$$W_T = \frac{k_T(s+1)}{3s+1}, W_u = \frac{k_u}{2s+1}, W_f = \frac{4}{4s+1} \quad (7)$$

In this case

$$W_g^\varepsilon = \frac{(3s+1)(2s+1)}{(3s+1)(2s+1)+k_T k_u (s+1)} \quad (8)$$

$$W_f^\varepsilon = \frac{1}{[(3s+1)(2s+1)+k_T k_u (s+1)](4s+1)}$$

$$\text{Then, } \Delta_s = \Delta_s^g + \Delta_s^f = \frac{1}{1+k_T k_u} - \frac{4}{1+k_T k_u} = -\frac{3}{1+k_T k_u}$$

Figure 2 shows the graph of the dependence of the static error regulator on the amplification factor. As can be seen from the figure, the amplification factor is already decreasing the static error.

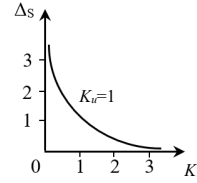


Figure 2. Dependence of the static error on the gain of the regulator

Figures 3a, 3b shows the corresponding regulation system (a) and dynamic error  $\varepsilon(t)$  at  $K_T = 20$  value (b).

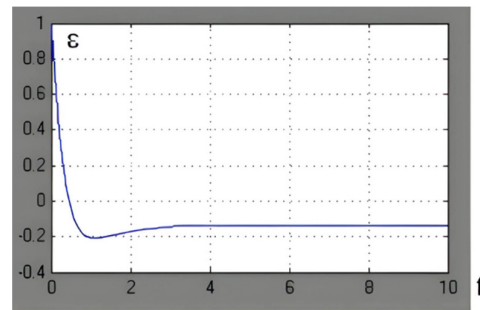
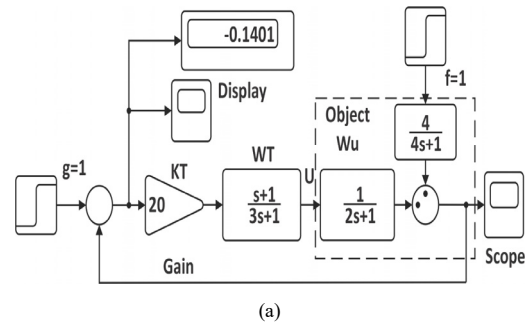


Figure 3. Regulation system (a) and dynamic error  $\varepsilon(t)$  at  $K_T=20$  (b)

As can be seen from the display,  $\Delta_s \approx \varepsilon(10) = -0.1401$ . Figure 4 shows the reduction of the adjustment error at values  $K_T = [2 \ 5 \ 10 \ 20]$  of the gain coefficient of the P-regulator for the aperiodic object  $\frac{dy}{dt} = -0.5y + u$ .

As it can be seen, the smallest static error was obtained at the value  $\Delta_s \approx \varepsilon(1) = 0.1025$   $K_T = 20$ . Increasing the  $K_T$  coefficient as much as possible, although not infinite in practice, leads to a decrease in the static error. Figures 5a, 5b shows the regulation scheme with the P-regulator and the switching characteristics of the amplification factor at  $K_T = [1 \ 10 \ 20]$  values.

As can be seen, as the value of  $K_T$  increases, volatility increases and stability reserves decrease. At the value of  $K_T=20$  there is already a violation of stability. At this time, as the value of the amplification factor increases, the oscillation increases and the stability reserves decrease. Therefore, the main issue is to find the maximum value of the amplification factor under the condition of the smallest static error.

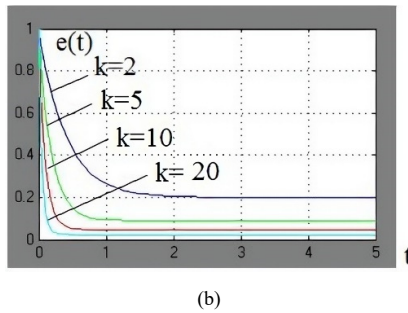
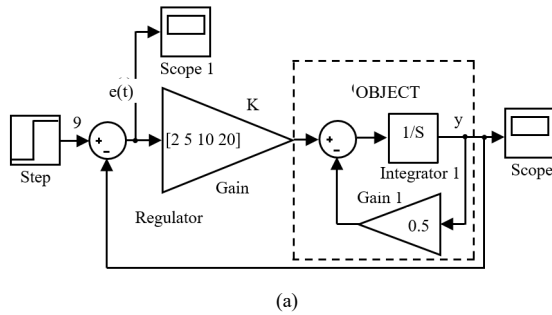


Figure 4. Dependence of the error on the gain of the regulator

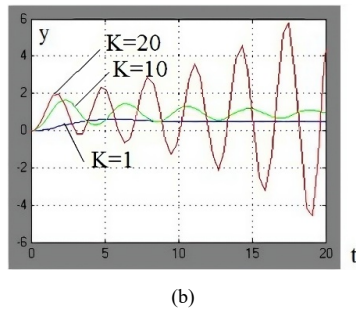
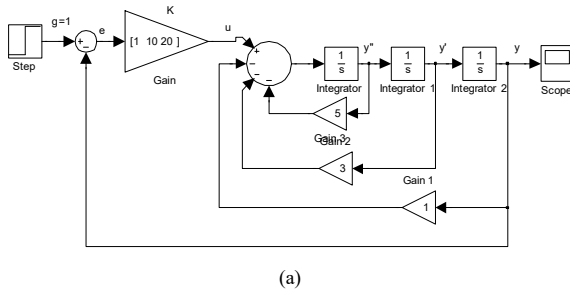


Figure 5. Regulation scheme and switching characteristics with a P-regulator

In this work, the regulation scheme and transition characteristics of the P-regulator have been established and it has been shown that as the value of  $K_T$  increases, the oscillation increases and the stability reserves decrease, at the value of  $K_T=20$ , the excess stability violation is observed.

## 2. MAIN PART

Invariant control systems against external excitations are considered more efficient in practical applications. G.V. Shipanov, who laid the foundation of the idea of

invariance, tried to obtain the condition of absolute invariance due to the internal relations of the system [1].

The following cases can be noted:

1. Infinitely increasing the amplification factor of the regulator and the observer does not lead to a violation of the stability of the system. It achieves high speed and accuracy. In many other works, methods of infinitely increasing the amplification factor of the open system have been proposed [4, 5].

2. Internal coordinates (state variables) of the systems are used to estimate the excitation effects. In addition, when the transfer function of the object changes, it is not necessary to rebuild the compensator due to the excitation effect channel. That is why, unlike the known methods, the system proposed in the work is called the second-order invariant system [6, 7, 8].

Let's look at object modeling [9, 10]:

$$\ddot{y} = -0.7\dot{y} - y + 4u + \tilde{\phi}(t)$$

$$f(t, y) = -0.7\dot{y} - y, b = 4$$

$$\tilde{\phi}(t) = \phi(t) + 0.4\dot{\phi}(t), \phi(t) = \sin(10t)$$

$$y_d(t) = 1(t)$$

The object's transfer function:

$$W_u = \frac{2}{s^2 + 0.7s + 1}, W_f = \frac{0.4s + 1}{s^2 + 0.5s + 1}$$

For compensator:

$$W_k = \frac{W_f}{W_u} = \frac{0.25s + 0.7}{0.02s + 1}$$

PID controller is used to ensure the invariance of the system:

$$W_p = \frac{k_p s^2 + k_i s + k_d}{s}$$

Figure 6 shows the modeling scheme in MATLAB/Simulink [9].

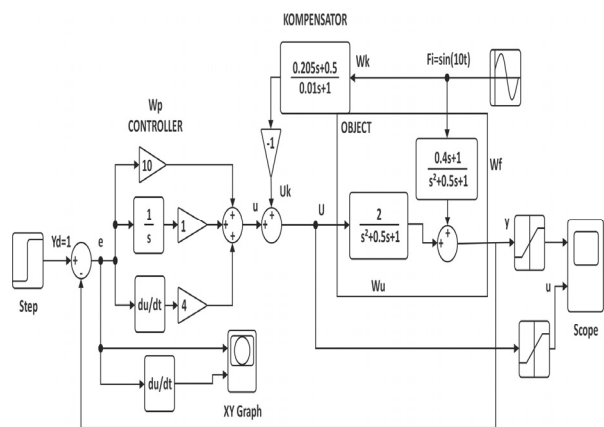


Figure 6. Modeling scheme in MATLAB/Simulink

Figure 7 shows the results of dynamic modeling. The tuning parameters were adopted as follows:

$$k_p = 10, k_i = 1, k_d = 4.$$

Thus, harmonics disappear in the output.

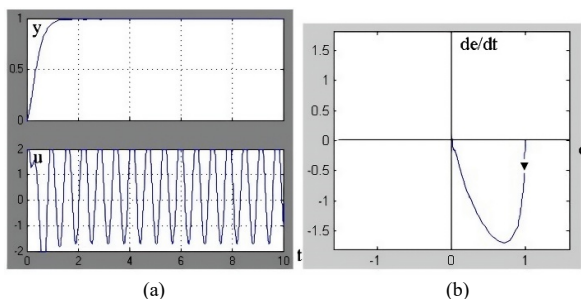


Figure 7. Dynamic characteristics of the system

Figure 8 shows the structural scheme of the proposed invariant system.

$$u_{eq} = -0.5(c_1 e + \ddot{y}_d(t) + F(t)),$$

$$u_{st} = -0.5k_p \times s(t), \quad c_1 = 4, \quad N = 42, \quad k_p = 16 \quad (14)$$

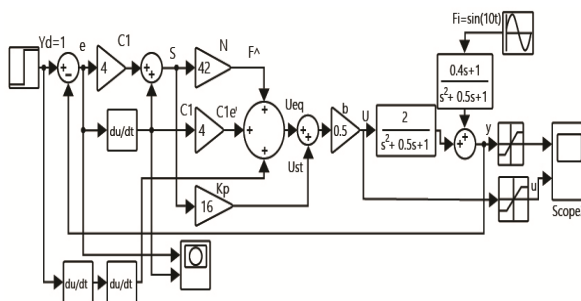


Figure 8. Structural diagram of the proposed invariant system

Figures 9a, 9b shows the dynamic characteristics of the system.

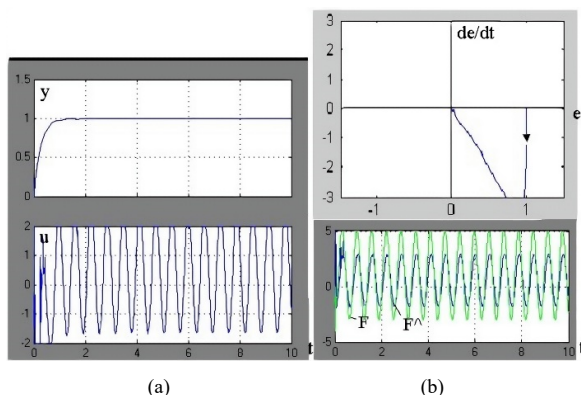


Figure 9. Dynamic characteristics of the system

### 3. CONCLUSIONS

Based on analytical and computer studies, the following main conclusions were obtained. The shortcomings of the existing management systems and algorithms were analyzed, and as a result, the issues to be considered were determined. A single-loop structure of the control system, which is the primary issue in the indicated direction, which allows to increase the amplification factor without breaking the stability, and a method of ensuring invariance against uncontrolled external exciting effects, is proposed.

The effectiveness and comparative analysis of the proposed methods and algorithms were evaluated by modeling and studying test problems in MATLAB/Simulink. The obtained positive results pave the way for the practical use of system technical analogs of Simulink schemes of synthesized control systems. As it can be seen, the quality indicators of the transition characteristics  $y(t)$  are almost the same, but in the second case, the external excitation effect is not measured.

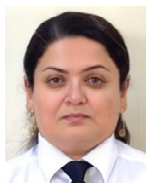
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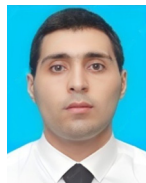
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