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ATTITUDES OF MATHEMATICS AND PHYSICS TEACHERS TOWARDS DIFFICULTIES IN MODELING WITH DIFFERENTIAL EQUATIONS IN SECONDARY SCHOOL

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Abstract- Modeling by differential equations is an essential competence required in mathematics and physics to perceive well many real problems. Several studies have shown that the acquisition of this competence by secondary school students (17-19 years old) is a task that poses several types of difficulties. In this work, we are interested in exploring the attitudes of teachers of mathematics and physics in secondary school towards the difficulties encountered by students in implementing modeling by differential equations. For this purpose, we conducted a survey through a questionnaire among teachers of the two disciplines. The results obtained show that the attitudes of teachers of both disciplines are not correlated to seniority in teaching. But clear correlations were observed, at the $p<0.05$ level of statistical significance, in teachers' attitudes towards the difficulties encountered in the different steps of the modeling. A principal component analysis was also performed, and it showed that the teacher's attitude towards the step of validating the results of solving the differential equations can account for more than 66% of the total variance.

Keywords: Modeling, Modeling Cycle, Difficulties in Modeling, Attitudes, Differential Equation.

1. INTRODUCTION

Since their genesis, differential equations, denoted in the sequel by DE, have become an essential tool for the study of many problems from various fields. An overview on this point can be obtained in [1], for example, and in the references therein. Thus, the study of DE has been included in secondary school curriculum, and many efforts have been deployed by researchers concerning the instruction of DE. Despite its importance and frequent applications, teaching and learning DE is still considered one of the most difficult, especially at the pre-university level as stated recently in [2] and [3]. Exploring efficient and innovative strategies for teaching differential equations had remained a focus in mathematics education and [4].

Historically, DE teaching has been influenced by various attempts to reform pedagogical practices that were undertaken at the beginning of the last century. Many authors have called for the use of problems that are actually interesting and relevant to students. At that time, applications were seen as a tool to enhance the learning process. They were used with the aim of concretizing the issues and motivating students, rather than preparing them to deal with real-life problems. But for Blum [5], mathematics learning should be sustained by connecting to real life. Students should learn to understand their environment and real-life situations with the help of mathematics and to develop general mathematical skills. In addition, this mutual transition between reality and mathematics is an essential prerequisite for being open to new situations.

The integration of teaching of mathematics through problems has led many authors to differentiate between application and modeling. In application, the activity focuses on moving from mathematics to the real-world context and mainly to products. In modeling the emphasis is on the complementarity of mutual transitions between reality and mathematics [6]. This does not imply that mathematics is unreal, but it is considered an area which provides tools for finding stable answers to questions posed in real situations.

Blum justifies the introduction of mathematical modeling in education by the following four types of considerations [7].

- Pragmatic: To understand and master real-world situations, it is necessary to establish a clear and strong link with relevant application and modeling cases.
- Formal: Strategic mathematical skills can also be developed by modeling exercises. In this respect, we can cite the example of mathematical reasoning which can be developed through admissibility checks. However, the appropriation of modeling competence can only occur by examining appropriately chosen practical and modeling situations.
- Cultural: Dealing with real-world situations through mathematical tools is crucial for developing a correct view on mathematics as a science in the broadest sense.

Indeed, modeling activities confer a cultural aspect to mathematics.

- Psychological: Involving examples from various fields can be a good factor in stimulating students' engagement with mathematics, demonstrating the appropriateness of the mathematical content and structuring it in a manner that maximizes comprehension.

In spite of the important place of modeling, classroom observations reveal little implementation of modeling in courses and class exams, as indicated in [8]. There are many factors that may be behind this situation. Modeling has been introduced in the curricula for a very short time. As a result, many teachers did not take advantage of the training to acquire the skills needed to teach modeling. Therefore, many teachers do not know how to handle modeling situations in their classroom or how to proceed when students are working on such situations. In 2022, the authors in [9] showed that adequate choice of cognitive activities involved in the learning of DE is a determining factor for the ease of transfer of learning on this notion. They observed several types of difficulties in secondary school students in solving certain differential equations from the field of physics.

In his study aiming to understand the difficulties and weaknesses of students in their learning of DE, Arslan [10] concluded that despite the fact that some students did quite well in algebraic solutions, they did not understand the related concepts well, and they had clear difficulties in relation to these concepts. This failure in DE learning has also been noted by Rowland [11] in a study conducted with students of first-year undergraduate engineering students. He concluded that few students seemed to perceive that the units of each term of first-order ordinary differential equations must be similar, or if they did, they do not succeed in applying this information when it is needed. In addition, not many students were in a position to determine the units of a proportionality factor in a basic equation.

In his thesis focused on differential equations as a means of mathematical modeling in secondary school physics and mathematics, Rodriguez [12] explored the types of tasks that students are asked to perform, and the kinds of techniques they are expected to use, when modeling situations using differential equations. The author found that the existing approach in the mathematics classroom was, most of the time, reduced to mathematical tasks only, such as solving differential equations of the form $y'=ay+b$, finding a particular solution that satisfies an initial condition, and sometimes studying the solution function. He also observed that contrary to the intention of the programs on the Physics-Mathematics interaction, students have difficulty using techniques learned in Mathematics class when setting up the experimental situation in Physics.

In a study, carried out with some students in a higher-level mathematics course, on the evidence about the resources that students use when establishing relationships between a contextual situation and an ordinary differential equation, Camacho-Machin and Guerrero-Ortiz [13] deduced that the difficulties in interpretation are due to the literal relationship that students establish between their mental model of the development of a phenomenon and its mathematical representations. Sijmekens and other authors [14] investigated the influence of the employment of contextualized situations in the instruction of differential equations on the capacity of engineering students to form and interpret differential equations. This study reveals that by providing sufficiently contextualized problems, students' skills in constructing and interpreting differential equations are enhanced. Furthermore, it has been pointed out that students' progress in these skills has no effect on their achievement of procedural knowledge.

The failures observed in all the previous studies, which concern several levels of education attest that modeling is a difficult competence not only to acquire by students but also to be implemented by teachers. Teaching mathematical modeling in the school environment is a cognitively challenging task as confirmed in many studies as [15] for instance. Hence, mathematics teachers should be empowered with different skills, disciplinary and non-disciplinary knowledge, task and teaching proposals, as well as appropriate attitudes and conceptions to deal with modeling in an appropriate way in class.

The literature on the issue of teacher obstacles repeatedly refers to the time factor. Teachers need more time to adapt tasks evoked by modeling to the needs of the classroom [16]. In addition, teaching is becoming more demanding. Teachers need additional skills to deal with this new approach of teaching, especially when the context is from a subject they have not studied. Performance assessment is also a problem, as teachers feel overwhelmed by the complexity of the modeling process.

It is also important to note that in studies conducted [17] and related to teachers' attitudes, it was revealed that teachers do not consider modeling as a mathematical activity. This situation prompts many researchers to think about how to promote the status of modeling in education contexts. For example, Winther [18] investigated the conditions of the implementation of modeling activities in physical science teaching and he showed that it is necessary that the students are well accompanied to help them overcome difficulties that arise during the establishment of a modeling process.

Given this situation, we focus in this work on the following problematic: how secondary school mathematics and physics teachers perceive students' difficulties in implementing the differential equations modeling process?

In relation to this problem, we set the following issues:

1. What are the attitudes of mathematics and physics teachers regarding difficulties in modeling with differential equations in secondary school?
2. Are there disparities between the attitudes of mathematics and physics teachers regarding difficulties in modeling with differential equations in secondary school?

2. THEORETICAL FRAMEWORK

In the literature on modeling and applications there are many different modeling cycles. The choice of one representation or another depends on the objectives of the analysis [7]. From a cognitive viewpoint, the seven-step process developed by Blum and Leib in [19] is considered to be particularly useful. It is a mixture of models from applied mathematics and, linguistics, and cognitive psychology. According to the cyclical scheme of Blum and Leib [19], the modeling process starts from a realistic situation, which involves an original problematic situation that is approached using mathematical resources. Afterwards, this situation is converted into a conceptual model in accordance with the modelers' background, experience, objectives and interests.

Thus, a mental portrait of the situation is produced which is the result of an individual perception of reality [20]. The resulting simplification and elucidation of the mental representation generates a real model [21]. This involves the identification of the problem, which consists of the choice between several possible models, and the experimental determination of the parameters that intervene as assumptions of the model. According to Henry, this task is quite important. In fact, he states that "if we want to introduce a real experimental approach in mathematics, we should not neglect the first step of modeling at the level of the concrete situation: the observation of the real situation" Henry [22].

Then a process of mathematization converts the appropriate items, relations and hypotheses of the real model into a mathematical representation, yielding a mathematical model that can be helpful in addressing the perceived problem [7]. Mathematical techniques are employed to solve the mathematical problem in the developed model and to achieve a result. Yet, modeling in mathematics is not restricted to moving a problem from reality to a mathematical problem, it also includes the reverse work. That is to say, to put in confrontation the mathematical reasoning and the reality. Consequently, the mathematical findings must be interpreted in the light of the context of the original real-life problem [23]. Afterwards, the whole process must be subjected to validation. This means the evaluation of the degree of approximation of the theoretical results, obtained with the corresponding experimental values, and the decision whether the model is well suited for the situation under study or not. If the chosen solution or procedure is not satisfactory, some steps or the entire process must be redone with a modified or totally new model. At the end, the solution to the initial real-world problem will be exhibited.

According to [7], the ability to carry out these steps is linked to some skills or sub-skills such as the correct perception of the given real situation or the explanation of mathematical results in relation to the situation studied. Models are not only aimed at description and explanation, but also at prediction and even creation of real-world elements. The capacity to perform each sub-process can be viewed as a sub- competency of modeling [24]. For Blum [7] modeling competence refers to the ability to build, use, or adapt mathematical models by performing the process steps in an appropriate manner, as well as analyzing or confronting given models. Modeling competence can then be understood as a mixture of several different sub-competences. In Table 1, we provide sub-competencies characterized by Greefrath and Vorholter [23] in accordance with the modeling cycle of Blum and Leib [19].

Table 1. Sub categories of modeling

Sub-competency	Explanation
Understanding	Students represent the problematic situation and form their special mental model. This allows them to acquire an understanding of the issue
Simplifying	Students distinguish between important and irrelevant data about a realistic situation
Mathematizing	Students convert simplified real-life situations into equations, figures, diagrams, functions, etc., thus forming a mathematical model
Working Mathematically	Students employ some heuristic approaches and use their mathematical background to solve the mathematical problem
Interpreting	The students transfer the results deduced from the model to the real context and thus obtain tangible results
Validating	Students examine the appropriateness of the actual findings in the situation model
Exposing	Students match the answers found in the model with the actual data and thus develop an answer to the main question

Taking into account the tasks in Table 1, a considerable amount of research has been undertaken on errors, obstacles or difficulties involved in the modeling processes. In particular, Klock and Siller [24] developed an interesting list, from a practical point of view, of difficulties related to each of the following five categories:

1. Developing a model of the actual world
2. Development of a mathematical model
3. Carrying out mathematical work.
4. Interpreting
5. Validating

3. METHODOLOGY

This research is of an exploratory type and aims to present as detailed as possible a description of the attitudes of mathematics and physics teachers regarding difficulties in modeling with differential equations. So, it is appropriate to use a mixed approach that combines qualitative analysis supported by the literature review carried out and quantitative analysis using statistical tools. To explore the attitudes of teachers, we chose to survey teachers of mathematics and physics who actually taught in final classes in science or technology in secondary school. In these classes, the mathematics and physics curricula stipulate that modeling by means of differential equations is a main skill that students must acquire [26, 27].

The survey is carried out using a questionnaire (Table 2) whose development of items was based on the conclusions summarized previously and which enabled us to identify the following hypotheses:

1. The teachers of the two disciplines do not have the same representations on the modeling steps using differential equations.
2. The teachers of the two disciplines do not have the same appreciation of the difficulties encountered by the students during the implementation of the modeling.

For each of the five steps listed at the end of the previous section, we have suggested a possible source of difficulties that students may encounter. Each possibility represents a variable denoted in the sequel by V_i (i=1,…,5).

Table 2. The questionnaire administered

Discipline taught	Mathematics				
	Physics				
Length of service as a teacher) by years)	Less than 5		Between 5 and 10		More than 10
Number of school years of teaching of scientific final classes	Less than 3		More than 3 and Less than 5		More than 5
List of possible difficulties related to each step of the modeling process	Strongly disagree	Not agree	Neutral	Agree	Strongly agree
1. In forming a model of the real world, students fail to identify relevant variables					
Students fail to represent the situations by a differential equation because they are unable to establish the dependencies between the variables.					
The difficulties encountered by students in solving the differential equation, representing the situation studied, are due to the use of inadequate strategies or algorithms					
In the interpretation step, students fail to identify the correct meaning of the solution of the differential equation					
Students fail to validate the formed mathematical model (ED) because they do not identify the influence of real-world constraints on the mathematical results					

To validate the content of the questionnaire, it was presented to three researchers in the field of mathematics education and then administered to six teachers, 3 of mathematics and 3 of physics, working in different secondary schools who had already teaching final classes (last class in secondary school). Following this pre-test, minor adjustments in the wording of certain questions were made to the questionnaire and then administered online in March 2023. The questionnaire was distributed online, with the help of some education inspectors, within group's teachers working in different schools at the two Regional Education and Training Academies of Rabat Sale Kenitra and Tangier Tetouan Al Hoceima.

The choice of an online questionnaire has the advantage of allowing the anonymity of the respondents and moreover, they feel assured that they will not be exposed to a direct judgment on their answers [28]. The total number of teachers who completed the questionnaire was 70, evenly distributed over the two disciplines (mathematics and physics). It is important to emphasize that the data collection was limited to those teachers who declared to be familiar with the modeling steps. Professional specificities of the participants in terms of seniority in teaching and the number of years they have been in charge of final classes are described in Table 3.

Table 3. Professional characteristics of the sample

	Seniority in teaching			Duration of teaching terminal classes		
Duration	Less than 5 years	Between 5 and 10 years	More than 10 years	Less than 3 years	Betwee n 3 and 5 years	More than 5 years
Mathematics teachers	3	16	16	10	3	22
Physics teachers	2	8	25	5	5	25

The answers to the questions are collected and coded according to a quantitative scale from 1 to 5 on a gradation between "Strongly disagree" and "Strongly agree" which corresponds to a Likert scale often used in research to measure attitudes and cognitive constructs. The choice of an odd scale also gives the respondent the possibility of positioning himself on a central response modality. The table of responses summarizes, for each individual in a row, the coded values of his or her responses to the questions, in columns. The objective is to analyze the table in order to identify the main design orientations that emerge from the set of responses, i.e., coherent sets of responses reflecting particular designs. We will carry out a multivariate statistical analysis to study the structure of the responses. The synthesis carried out makes it possible to highlight the redundancies or the possible correlations between questions, for which we obtain globally similar (positive correlation) or dissimilar (negative correlation) answers [29]. Data processing is performed using SPSS software.

4. RESULTS

4.1. Univariate Descriptives

In order to get a first view of the population of respondents, we analyzed the responses using simple descriptive statistics.

Table 4. Descriptive data

	Discipline	N	Mean	Std. Deviation	Std. Error Mean
V_1	Maths	35	3.1142	0.86675	0.146
	Physics	35	3.0857	1.26889	0.214
V_2	Maths	35	3.3142	0.99325	0.167
	Physics	35	3.1714	1.12421	0.190
V_3	Maths	35	3.1714	0.98475	0.166
	Physics	35	2.9428	1.10992	0.187
V_4	Maths	35	3.1140	1.0784	0.182
	Physics	35	3.3140	1.3234	0.223
V_5	Maths	35	3.4570	1.0387	0.175
	Physics	35	3.200	1.2078	0.204

It is interesting to note that these results show that the means of each variable are almost the same for both groups of teachers. However, we should not rely directly on this appearance. Let us then carry out a test of the means using the t-test of two independent samples.

4.2. Comparison of Means

We recall that *t*-test is a statistical test employed for comparing the means of two distinct groups. When the significance level is small ($p<0.05$), we can refuse the hypothesis that the two groups come from the same population and conclude that the two means do not refer to the same population. To undertake the *t*-test, it is necessary to assess the homogeneity of variances for the samples. This can be given by the use of Levene's test.

Table 5. Independent samples test

		Levene's Test for Equality of Variances		*t*-test for Equality of Means				
		F	Sig.	*t*	*df*	Sig. (2-tailed)	Mean Difference	Std. Error Difference
V_1	Equal variances assumed	6.392	0.014	0.110	68	0.913	0.0285	0.2597
	Equal variances not assumed			0.110	60.05	0.913	0.0285	0.2597
V_2	Equal variances assumed	0.457	0.501	0.563	68	0.575	0.1428	0.2535
	Equal variances not assumed			0.563	66.98	0.575	0.1428	0.2535
V_3	Equal variances assumed	0.537	0.466	0.911	68	0.365	0.2285	0.2508
	Equal variances not assumed			0.911	67.04	0.365	0.2285	0.2508
V_4	Equal variances assumed	3.846	0.054	-0.693	68	0.491	-0.2000	.2886
	Equal variances not assumed			-0.693	65.33	0.491	-0.2000	0.2886
V_5	Equal variances assumed	1.354	0.249	0.955	68	0.343	0.2571	0.2693
	Equal variances not assumed			0.955	66.51	0.343	0.2571	0.2693

4.3. Bivariate Analysis

The cross-tabulation of the variables related to the modeling steps by the differential equations allowed to highlight some significant correlations, at the 0.01 level (2-tailed), as it is shown in Table 6.

In view of these results, which reveal certain correlations between the variables studied, we can think that it would be interesting to further process the data. To do this, we have carried out a principal component analysis (PCA).

4.4. PCA Analysis

In order to highlight the different attitudes of teachers according to the representations that they let appear in their answers to the questionnaire, we carried out a principal component analysis which allows a multivariate analysis of all the variables. A PCA can only be implemented with quantitative variables or with hierarchical variables measured, for example, using a Likert scale. The principle of PCA is to minimize the number of variables. The new variables are called factors. The factors are linear functions of the initial variables.

In PCA, the adequacy of the sample must first be examined. To do this, two tests can be administered: the Kaiser-Meyer-Olkin (KMO) test and Bartlett's test of sphericity. The first gives a proportion of the variance between variables that could be a common variance. It is scored from zero to one, with zero being inappropriate, while a value close to one is appropriate. For the Bartlett test, the observed correlation matrix is compared to the identity matrix. In general, KMO values of at least 0.50 and $p<0.05$ for the Bartlett test of sphericity are considered acceptable.

Table 6. Correlations between variables

		Seniority	Years of teaching in terminals	V_1	V_2	V_3	V_4	V_5
Seniority	Correlation Coefficient	1						
	Sig. (2-tailed)	.						
	N	70						
Years of teaching in terminals	Correlation Coefficient	0.561**	1					
	Sig. (2-tailed)	0	.					
	N	70	70					
V_1	Correlation Coefficient	-0.074	0.125	1				
	Sig. (2-tailed)	0.544	0.301	.				
	N	70	70	70				
V_2	Correlation Coefficient	0.059	0.210	0.566**	1			
	Sig. (2-tailed)	0.628	0.081	0	.			
	N	70	70	70	70			
V_3	Correlation Coefficient	-0.057	0	0.462**	0.666**	1		
	Sig. (2-tailed)	0.638	1	0	0	.		
	N	70	70	70	70	70		
V_4	Correlation Coefficient	0.143	0.051	0.495**	0.470**	0.606**	1	
	Sig. (2-tailed)	0.237	0.675	0	0	0	.	
	N	70	70	70	70	70	70	
V_5	Correlation Coefficient	0.071	0.113	0.541**	0.551**	0.740**	0.673**	1
	Sig. (2-tailed)	0.558	0.351	0	0	0	0	.
	N	70	70	70	70	70	70	70

Table 7. KMO and Bartlett's test

KMO		0.825
Bartlett's Test of Sphericity	Approx. Chi-Square	169.361
	df	10
	Sig.	0.000

We can then conclude that:

- Since the KMO index is greater enough than 0.5, so all the items are factorable.
- The Bartlett test revealed that the calculated p-value is below the 0.05 level of significance.

Therefore, the hypothesis that there is no correlation significantly different from 0 between the variables should be rejected and the fact that there are correlations that are not all equal to zero should be retained. Concerning the reliability, calculation of the Cronbach's coefficient was performed in both cases, for all the items and considering only the variables V_i. We find the results in Table 8.

Table 8. Cronbach Alpha

Cronbach's Alpha	N of Items
0.807	7
0.872	5

Hence, the reliability of our questionnaire is quite satisfactory. Thus, all the items contribute to the reliability of the questionnaire and no purification is needed. By specifying that the PCA is run on all items without fixing previously the number of factors requested, we obtained the results reported in Table 9.

Table 9. Total variance explained

Component	Initial Eigenvalues			Extraction Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	3.322	66.441	66.441	3.32	66.441	66.441
2	0.602	12.046	78.488			
3	0.501	10.011	88.499			
4	0.354	7.087	95.586			
5	0.221	4.414	100.0			

Table 10. Component matrix

	Component
	1
Variable 1	0.728
Variable 2	0.811
Variable 3	0.855
Variable 4	0.797
Variable 5	0.876

Under the Kaiser criterion (the eigenvalue is more than or equal to 1), only components with an eigenvalue greater than 1 are retained. Hence, factor 1 explains 66.441% of the total variance. With respect to this factor, the component matrix is as follows.

5. DISCUSSION

The results obtained show, first of all, that the averages are almost the same among the mathematics teachers and their physics counterparts for all the variables studied, namely the difficulties related to the modeling steps by differential equations. This means that the two groups share in some way the same attitudes regarding these difficulties. In addition, the deviations from the averages are also roughly equal. This result prompted us to perform a test for the means. Note that the degrees of freedom are high. This implies that the test performs well. Secondly, it reveals the following two cases. The first is manifested by the equality of variances for the two categories of teachers, which corresponds to a value of Levene's test with a p-value lower than 0.05, the value of t does not allow the hypothesis of equality of averages to be rejected. The attitudes towards the first step of modeling by differential equations are part of this first case, i.e. teachers of both disciplines share the same attitude on the fact that students are unable to identify the adequate variables in the formation of a model of the real world. However, the averages for this variable are close to 3. This means that both groups are neutral with respect to this hypothesis.

The second case is where there is a difference in the variances of the two groups according to the values indicated by Levene's test. Here again, it is clear that there is no difference in averages that are significantly close to 3. This is interpreted by the fact that the two groups tend to be undecided about the difficulties encountered by the students in implementing differential equation modeling. This state of neutrality can be explained by many facts. On one hand, teaching mathematical modeling in the classroom is cognitively challenging as stated by many authors like [15], and [30] for instance. On the other, teaching modeling requires enough time to perform such tasks as observed by [16].

In studying the correlations between the different variables, the following remarks can be made from Table 6:

- Contradictorily, there is no confirmed correlation between the number of years spent in service and the attitude on the difficulties that can emerge naturally in the process of modeling by the DE. This is more surprising when the same remark extends to the character of decorrelation which marks the number of years of practice with the final classes and the management of the difficulties noted with the students.
- Positive correlations are quite clear between the different V_i variables. Sometimes this correlation is moderate as in the case of the attitude that the students fail to identify relevant variables in forming a model of the real world and all other variables. This is also the case for the attitudes on the fact that the students fail to represent the situations by a differential equation because they are unable to establish the dependencies between the variables with the difficulties that may arise in the steps of interpretation of the results from the resolution of the DE or during the validation of the model constructed.

This last second point prompted us to implement a PCA with the aim of confirming the correlations observed in the bivariate analysis and, above all, to try to determine the principal factors that explain the variability in our sample. It should be noted that the verification of the statistical conditions necessary to carry out the PCA led to fairly satisfactory results for the KMO and Bartlett's Test. The striking result in this analysis, carried out without predefining the number of factors required, was that one principal component was responsible for more than 2/3 (about 66.441%) of the variance observed.

Referring to the results in Table 8, we see that the fifth variable 5 is the best represented according to the principal component. Therefore, as an interpretation of this principal component, we can say that the attitude that the students fail to validate the formed mathematical model (DE) because they do not identify the influence of real-world constraints on the mathematical results is responsible for the majority of the variances. This also confirms the fact that the values of the correlation coefficients of V_5 with the other variables take the higher values in Table 5.

6. CONCLUSION

Modeling in the field of education has long become an essential asset for improving the learning of mathematics, and also for better understanding real phenomena. Under this vision, several efforts have been made to promote the acquisition of this essential skill. In particular, modeling by differential equations has taken a good part in the research work. Based on observations of difficulties in implementing the modeling process in teaching practices and learning processes, several authors have focused on the development of sub-competences related to modeling [24]. Consequently, the identification of the difficulties encountered by students has become more operational [25]. To understand better, the problems that hinder the proper functioning of differential equation modeling in the two disciplines, mathematics and physics, in secondary school as pointed out in [9], we aimed in this study to explore the attitudes of the teachers of the two disciplines towards the sources of difficulties related to the different modeling steps.

The questionnaire conducted among a random group of classroom teachers of both disciplines led to the following conclusions. Generally, seniority in teaching had no effect on the attitudes of teachers of both subjects to the modeling question. The teachers surveyed showed significantly correlated responses for all five modeling steps. For the fifth step, which deals with the validation of the mathematical model by subjecting it to evaluation under real-world constraints, it was found to be the most correlated with the attitudes of the other four steps. It follows that the attitudes of the mathematics and physics teachers can be summarized by the responses obtained on the attitudes on the last validation step.

In order to go further in this direction, based on the grid developed by Klock and Siller [25] on the difficulties related to the sub-competences of modeling, we intend in the future to conduct research on the following two questions:

1. What attitudes do mathematics teachers have towards these difficulties in DE modeling?
2. What difficulties can be observed in secondary school or university students when analyzing their productions via this grid implemented in DE modeling situations?

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