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DIFFICULTIES IN PROBLEM SOLVING AND UNDERSTANDING SEMIOTIC REGISTERS OF NUMERICAL FUNCTIONS CONCEPTS

D. Bouanfir [1] M. Abid [2] B. El Wahbi [1]

1. Laboratory of Geometric Analysis and Applications, Faculty of Science, Ibn Tofail University, Kenitra, Morocco
douaa.bouanfir@uit.ac.ma, belwahbi@yahoo.fr
2. Laboratory of Scientific Research and Pedagogical Innovation, Regional Center for Education and Training Professions, Madinat Al Irfane Rabat, Morocco, profabidmed@gmail.com

Abstract- The present study aims to identify and compare difficulties in numerical function solving problem and realizing registers semiotic representation related to this concept encounter by Moroccan second-year baccalaureate in high school. The sample consisted of 62 pupils, equally divided into two groups: physics chemistry (PC) and mathematical sciences (MS). Pupils in both groups were asked to answer a supervised test consisting of a problem and an exercise (addressed only to MS group). Excel and SPSS software were used to examine and analyze the data obtained. The results demonstrated that, for the problem test, MS pupils' groups obtained a mean score 12.33 better than that of PC one 10.08. In fact, the PC group had greater difficulty in solving the problem, particularly in drawing the table of variation and studying infinite branches to produce the graphical representation (drawing the treated function, the restriction function and the reciprocal function of the restriction function). In addition, PC pupils have greater difficulty than MS pupils in realizing and converting between registers. For the exercise test, the MS group encountered difficulties in the logical processing and requiring the precision of the analytical characteristics of function graphs, they also have difficulties with the procedural knowledge (in the case of an unguided open question) illustrated in the conversion between registers. The outcomes of this study reveal that when pupils resolve a problem with a complex numerical function, they apply the quasi-automatic processes used in previous problems with adaptation to the current situation. However, they run into problems with treatment (derivation, limits, etc.) and realizing semiotic representations of this function at various points.

Keywords: Numerical Function, Register of Semiotic Representation, Mental Activity, Solving-Problem.

1. INTRODUCTION

The function concept is regarded as one of the most important and basic in all of mathematics. Mathematics which is referred generally as science language [1]. Indeed, elements such the point, the line and the plane were the fundamental constitution of Euclidean geometry from the days of ancient Greece right up to the modern era. In addition, the notions of function and derivative have formed the basis of the theory of mathematical analysis ever since [2]. However, the introduction of this concept into school curricula is essential from nursery school to university level [3]. In the Moroccan context, the official mathematics programs include the concept of function, following a didactic transposition from primary, middle and high schools to university (grid, enlarging and reducing shapes, proportionality, literal expressions, image point by translation, linear function, variable, antecedent, image, $f(x)$ notation, affine function, rational power functions, reciprocal functions, derivation, limit, primitive, logarithm, and exponential functions) [4].

Many efforts have been made by Morocco to improve the quality of education [5]. The Moroccan reform of mathematics curricula at the elementary and secondary levels is a component of a constructivist and socio-constructivist spiral approach that builds on students' previous knowledge and emphasizes problem-solving and student-centered learning. The spiral approach aids in the gradual and integrated development of students' personal competences, including knowledge, skills, attitudes, and values. Nevertheless, Moroccan students in grades 4 and 8 did not perform well in the international TIMSS assessment in mathematics [6]. Learners' difficulties in understanding mathematics in particular functions may be linked to the abstract nature of the concept and its multiple semiotic representations, as well as to the teaching approaches used. Duval [7] stated that Mathematical concepts are only examined through their semiotic representations, this means that it is difficult to understand and learn a mathematical concept without the use of the representation, because any given its representation cannot, however, fully describe it, since it reveals information about only some of its facets.

On other hand, Kaput [8] argues that translation between representational systems is a source for the construction of mathematical meaning while Gagatsis and Shiakalli [9] and Yerushalmy [10] confirm that the ability to move between representations of the function concept

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is well correlated with successful problem-solving. For that, we aim through this paper to explore some learning difficulties of numerical functions collected with the help of a performance test of learners of the 2nd year baccalaureate of the Moroccan high school based on the theory of the registers of semiotic representation [11]. This theoretical framework was chosen because it allows linking the understanding of mental activities and the procedural knowledge of the study of a concept with the ability to make transformations between its semiotic representations.

2. PROBLEMATIC OF STUDY

The study of numerical functions requires the investment of a set of skills in algebraic reading, comprehension, data analysis, numerical calculation, and geometric interpretation of numerical results. The acquisition of these skills facilitates the knowledge of the prerequisites of the study of numerical function: calculation of the definition domain, calculation of limit, calculation of derivation, the study of function variation and graphical representation [4]. But the absence of these tools makes the study of a function very complex and difficult. Ponte [2] indicated that learners arrive at high school with difficulties in processing graphs and algebraic expressions while Duval [12] concluded that to understand the function concept, it is essential to know their registers of semiotic representations and to convert the representations produced in one system into that of another system.

Our operational approach uses the didactic framework to assess the learning of the concept of numerical function acquired by students of the 2nd year Moroccan baccalaureate in experimental sciences of physics chemistry section (PC) and mathematical sciences section (MS). This approach is based on the following research questions:

- How well do pupils in the second year of the Moroccan baccalaureate sciences in solving the numerical function problem and realizing the registers of semiotic representation associated with this concept?
- Is there a statistically significant distinction in the PC and MS pupil groups' performance on the Moroccan baccalaureate's second year in terms of their mastery of problem-solving skills and their ability to convert between the registers of semiotic representation associated with the concept of a numerical function?

3. CONCEPTUAL ELEMENTS

3.1. Semiotic Register

The term semiotic framework is said when the work is done on a problem related to a specific theory or conceptual field [13]. To study a problem, the points of view and the rules of mathematical processing present registers. Duval [14] confirms that a mathematical object cannot be manipulated directly but through its registers of semiotic representation: the representations of the object (algebraic, figurative, and graphic) and the registers of natural language.

Duval [11] insists on the importance of exploiting the registers of representation for the understanding of mathematical concepts. Rogalski [13] reported that studying a mathematical object requires changing a point of view about that object by changing register of representation, this change in point of view is difficult for students knowing that working within one register is not conducive to learning as well as moving between multiple registers is essential for understanding a mathematical object. Understanding a mathematical object requires working with multiple registers of representation [12, 15], it is measured by the ability to do the processing and conversions between the registers of representation associated with the concept [16].

According to Duval [11] conversion activities have a crucial role in the progression of learning and understanding of a mathematical concept, he also notes that among the main objectives of conversion is to provide a new register considered as a guide to perform processing in another register. In the case of the study of numerical functions, the mathematical processing is done using the conversion between algebraic symbolic register (ASR), Table of variation register (TVR), Natural language register (NLR) and Graphic register (GR), in the semiotic chain of Figure 1 [13].

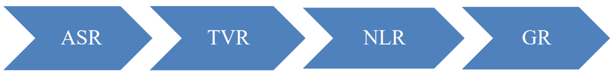

Figure 1. Semiotic chain associated with a numerical function [13]

The transition between two registers is done according to specific rules of conversion without any transfer of properties from the initial register to the incoming register because each register has its specific properties and treatments [13]. McGee and al. [17] note that in the case where there are n representations of a concept, there are n-1 conversions and treatment in the associated semiotic chain and $n!$ conversions and treatment between these representations in the totality.

3.2. Problem-Solving Skills in Mathematics

Solving a problem can be seen as a process consisting of four constant interaction phases. Pupils' ability to solve mathematical problems can be characterized as their capacity to understand problems, formulate solutions, implement the chosen strategies, and then re-evaluate the solutions to find new ways of solving or developing the problem [18]. Rittle and al. state that problem-solving skills are the set of steps to be followed to solve a specific type of problem. The practice of mathematical activities is developed through the teaching of mathematics, and this practice includes the practice of problem solving. At the same time that it should be prioritized in the teaching of mathematics, problem-solving is a fundamental ability that pupils should learn [19]. This method incorporates questions-whether they are asked by the teacher, the student, or the students themselves-along with the student's own action.

3.3. Mental Activity

Mental activity is an activity that requires a higher level of knowledge, for example, Duval [11] considered that semiotic representations allow the individual to externalize his mental representations in order to make them visible to others. The understanding of the concept of function requires overcoming some main epistemological obstacles through the corresponding mental actions [20]. The mental activities tested in this research are based on a transformation of the initial data of a representation within a register to obtain terminal data (these are internal treatments to a precise register).

3.4. Numerical Function

The term function is used for the first time by Leibniz (in 1673) who considered the function as a dependence of geometric quantities such as subtangents concerning the shape of a curve, it assures a principal place in the mathematical thought by its association with the study of the analytical expressions [2]. The notion of numerical function is useful for describing many different types of situations by bridging to geometry using the Cartesian representation [2]. It is a correspondence between variables that has great importance in the design and study of mathematical models [2]. The nature of multiple representations of the concept of function is a primary characteristic that makes it important in the learning process [21]. Studying functions allows students to make connections between representations of functions: symbolic (equations), numerical (tables), verbal (descriptive), and graphical (the graph of a function) [21].

3.5. Pedagogical Engineering of Education in Morocco

There are two major cycles in the pedagogical engineering of the Moroccan educational system. Pupils aged six years old complete six years of primary education during the first cycle. In the secondary cycle, pupils first enroll in a middle school (3 years) before moving on to a high school (3 years). Pupils attend both sorts of schools for six years before taking the National final baccalaureate exam [4]. Study in high school lasts three years, divided into two cycles: the common core, in which study lasts one year, after which Scientifics pupils are directed to the baccalaureate cycle (consisting of two years), which contains three sections: experimental sciences, mathematical sciences, and industrial sciences and technology [4].

The Moroccan curriculum allocates 7 and 5 hours per week, respectively, for the mathematical sciences and experimental sciences sections, to the teaching of mathematics during the baccalaureate cycle [20]. Mathematics education in the high school builds the intellectual mentality of young citizens [4]. However, the curriculum learning mathematics in the high school considers the study of numerical functions as a central study [22]. Understanding this concept is important, especially for students who want to continue their higher education study in the field of mathematics and engineering sciences.

3.6. Knowledge and Skills Required in Function Topics

We outline in Table 1 the objectives and prerequisites related to the function themes according to the Moroccan official Baccalaureate 2nd year curriculum, specialty Physics-Chemistry and Mathematical Sciences [4].

Table 1. Objective and prerequisites of function course in the 2nd-year Moroccan Baccalaureate [4]

Objective	Prerequisites
- Know the continuity of a function at a point, the continuity of a function on an interval and the whole part function. - Determine the image of an interval by a continuous function. - Know the intermediate value theorem, the continuity of the composite of two functions and the reciprocal function of a continuous and strictly monotonic function on an interval. - Creating the curve of a reciprocal function. - Know the function $x \to \sqrt[n]{x}$ with $(n \geq 1)$. - Know the limit of the continuous composite of two functions, the extension by continuity and the usual reciprocal function $x \to \sqrt[n]{x}$ and $x \to \arctan(x)$. (Also, in the Mathematical Sciences). - Know the derivative number, the derivative function, differential writing, the derivative of the composite of two functions, the derivative of the reciprocal function and the primitive functions of a function. - Study polynomial functions, rational functions, irrational functions and trigonometric functions. - Know the derivative function of the usual functions $x \to \sqrt[n]{x}$ and $x \to \arctan(x)$. (Also, in the Mathematical Sciences) - Studying the neperian logarithm function. - Know the usual logarithmic limits, the logarithmic derivative of a function, the logarithm function of base a and the decimal logarithm function. - Studying the neperian exponential function. - Know the usual exponential limits, the derivative of the exponential function and the exponential function of base a. - Know Rolle's theorem, finite increment theorem and finite increment inequality. (Also, in the Mathematical Sciences) - Know the differential equations as $y' = ay + b$ and $y'' + ay' + by = 0$ with $a \in \mathbb{R}$ and $b \in \mathbb{R}$. - Verification that a function f is the solution of these differential equations.	- Generalities of numerical functions (definition domain of a function, graphical representation of a function, equality of two functions, graphical interpretation of an even and odd function, studying the variations of a numerical function) - Studying the parabola of equation $y = ax^2$ $(a \neq 0)$ - Studying the hyperbola of equation $y = \frac{a}{x}$ $(a \in \mathbb{R}^*)$ - Graphical representation of the functions that lead to the usual functions $x \to -f(x)$, $x \to f(x) + a$ $(a \in \mathbb{R}^*)$ and $x \to f(x+a)$ $(a \in \mathbb{R}^*)$ - General information on numerical functions (Reminder and novelty). - The graphical representation of the function $x \to E(x)$. (Also, in the Mathematical Sciences A and B) - Limit of a numerical function. -Derivation -Graphical representation of a numerical function (asymptotic lines, parabolic branches, concavity of a curve and inflection point).

4. MATERIALS AND METHODS

4.1. Research Design

We employed a quantitative and analytical methodology in this study to gather and analyze data from each respondent. Before sharing the test with the participants, it was performed. This study set out to find second-year baccalaureate students in Morocco who had difficulty converting across semiotic registers when addressing problems related to the concept of function. This design enables us to compare the level of semiotic register acquisition when processing numerical functions at the high school and the degree of mastery of the problem-solving method among sections pupils.

4.2. Target Population

The study sample consisted of 62 Moroccan 2nd-year Baccalaureate students from three high schools in the Casablanca city during the 2020-2021 school year. Table 2 provides a detailed breakdown of the study sample.

Table 2. Distribution of the study sample

Groups	N	Percentage	Girls Number	Boys Number
PC	31	50%	17	14
MS	31	50%	18	13
Total	62	100%	35	27

4.3. Instrument of Research

The data for the study was obtained using a performance test which assessed the knowledge and skills of second-year Moroccan Baccalaureate students in the PC and MS sections about the conversion between semiotic representation registers.

This test consists of a problem concerning the realization of the study of a complex numerical function $(h(x)=\sqrt{2x+1}-\frac{x}{\sqrt{2x+1}})$ addressed to both PC and MS students, and an exercise aimed at identifying with a justification the curve that represents the primitive of a derivable function (the function curve is given in the statement) from three proposals of graphical representation of which the tangent exists at the point of specific abscissa for each one (addressed only to MS students). Each question in the suggested problem and exercise corresponds to the accomplishment of a mental and procedural activity, as indicated in Table 3.

4.4. Test Procedure

The sampling technique adopted is the voluntary participation of the learners in this research. They were informed that their response to the diagnostic test will be treated confidentially and anonymously and that the test is the subject of research. This performance test was first shared with 4 teachers with long experience in high school education to examine its reliability. This test was in the form of a supervised test for 45 minutes for PC students (only problem) and MS students (problem and exercise) knowing that each exercise is marked out of 20 points.

4.5. Data Analysis

The collected data were processed in Excel and IBM SPSS version 25 software. Descriptive and inferential statistics were carried out to analyze the data. All results were interpretively reported at $p < 0.05$.

Table 3. Treatments and conversions activities of each question of the proposed problem study test

	item	Treatments and conversions activities
Problem	Q_1. Determine D_h with justification.	Concept of the domain of definition and limit of a function: use the algebraic notation necessary to give the domain of definition of the function and treat the calculation of the limit in order to express the algebraic symbolic register "ASR".
	Q_2. Calculate the limits of h at the bounds of D_h.	
	Q_3. Draw the table of variation of h.	Concept of the table of variation: use the necessary conditions of the table of variation to express the table of variation register "TVR"
	Q_4. Studying the infinite branches.	Concept of infinite branch: use the conditions adopted to the function to make treatments with the symbolic algebraic register " ASR " and express the natural language register "NLR"
	Q_5. Draw in the same frame the curves C_h, C_g and $(C_{g^{-1}})$.	Knowing the previous concepts and the concept of a tangent, inflection point, function restriction, and reciprocal function to express the graphical register "GR" with specific processing steps.
Exercise	We give the curve C_f of a function f defined and derivable on $\mathbb{R}$. We give the representation of the three curves with the tangent in the dotted line of each at the point of abscissa 0. Knowing that the derivative number of a function g at a point A of abscissa a is the directing coefficient of the tangent to C_g at point A of abscissa a. Find, with justification, the curve that represents the primitive of f (Figure 2).	-Concept of primitive of a function: draw the table of variation of the primitive of f to eliminate C_3 and express the table of variation register "TVR". -Concept of directing coefficient of a tangent: use the directing coefficient of the tangent of C_1 and C_2 to eliminate C_2 and do treatments with the symbolic algebraic register " ASR ".

5. RESULTS

5.1. Student Performance on the Problem Test

5.1.1. Descriptive Analysis of the Sample

Table 4 summarizes all the information gathered from 62 pupils (*PC*+*MS* groups) in our sample regarding the problem-solving results. The sample's mean score was 11.20 with a standard deviation of 3.67; the lowest and highest scores were 4/20 and 20/20, respectively. According to the obtained results, the MS group had a mean score of (M_{MS}=12.33) with a standard deviation of 3.00 whereas the *PC* group acquired a mean score of (M_{PC}=10.08) with a standard deviation of 3.96.

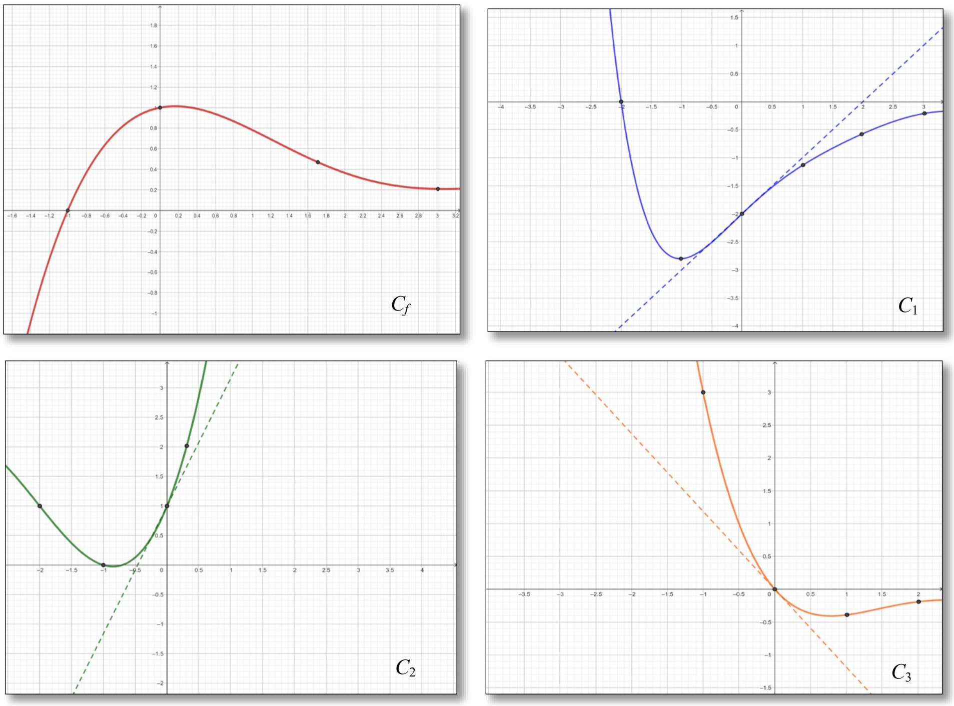

Figure 2. Curves of C_f, C_1, C_2 and C_3 subject of the exercise

The first overall reading of these results indicates that the pupils of the sample have difficulties in solving problems involving the processing of a complex numerical function. These relatively low means scores are due to the existing difficulties at Q_2 level (difficulty in treatment of the limit of the h function in the vicinity of $+\infty$), at Q_3 level (difficulty in treatment of the table of variation), at Q_4 level (difficulty in treatment of the limit of $\frac{h(x)}{x}$ in the vicinity of $+\infty$) and at Q_5 level (difficulty in treatment of the graphical representation of the function).

Table 4. Descriptive statistics based on problem score

Groups	N	Mean score	Minimum	Maximum	Std. Deviation
PC	31	10.0806	4.00	16.75	3.96410
MS	31	12.3306	6.75	20.00	3.00928
PC+MS	62	11.2056	4.00	20.00	3.66992

Figure 3 shows the percentage of the studied sample (PC and MS groups) that succeeded in achieving the semiotic registers. The Mean score corresponding to the result of the performance of solving-problem in this study (Table 4), does not reflect the degree of realization of the registers of representation indicated in Figure 3. In other words, because the processing was incomplete, the realization of the registers does not necessarily imply that the student received a mark/score. The obtained results show that all the pupils of the sample passed the ASR registers of representation even if there is a treatment difficulty in the limit of the function h in the vicinity of $+\infty$. On the other hand, pupils' performance in the other semiotic registers, has decreased. TVR and NLR registers were answered correctly by 84% and 73% of the learners, respectively, even if there are other students who have difficulty of treatment in the table of variation and the limit of $\frac{h(x)}{x}$ in the vicinity of $+\infty$ but GR register was answered successfully by only 23% of the pupils who participated while the rest of the pupils haven't made a graphical representation of the function h.

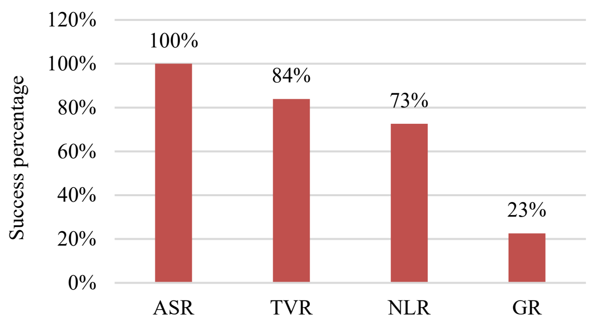

Figure 3. Percentage of sample success in realizing of semiotic representation registers

5.1.2. Comparative Analysis between PC and MS Groups

5.1.2.1. Comparison between Groups and Problem Score

The available data were normally distributed for the realization of numerical function study for both PC and MS streams. Indeed, for the PC group, the Kolmogorov-Smirnov p-value is 0.2 which is higher than 0.05 and the Shapiro-Wilk p-value is 0.079 which is greater than 0.05. Similarly, for the MS group, the Kolmogorov-Smirnov p-value is 0.057 which is superior to 0.05, and the Shapiro-Wilk p-value is 0.106 which is higher than 0.05 (Table 5).

Table 5. Normality test for the PC and MS samples groups in solving,
*. This is a lower bound of the true significance
a. Lilliefors Significance Correction

Groups	Kolmogorov-Smirnov [a]			Shapiro-Wilk		
	Statistic	*df*	Sig	Statistic	*df*	Sig.
PC	0.107	31	0.200 *	0.939	31	0.079
MS	0.155	31	0.057	0.944	31	0.106

Table 6 indicates that the significant value of Levene's test for homogeneity of variances is 0.062 which is greater than 0.05, so the variances of the PC students are equal to the variance of the *MS* students. The significant value of the student's t-test is equal to 0.015 which is lower than 0.05, so there is a significant distinction between the average score of the *PC* and *MS* groups in the numerical function problem–solving. As the mean score M_{MS}=12.33 > M_{PC}=10.08, we conclude that the performance in problem score of the *MS* group is greater than that of the *PC* one.

Table 6. Student's t-test for sample groups in solving problem

	Levene's Test for Equality of Variances	t-test for Equality of Means		
	Sig.	*T*	*df*	Sig. (2-tailed)
Equal variances assumed	0.062	-2.517	60	0.015

5.1.2.2. Comparison between Groups and Realization of Semiotic Registers

Figure 4 compares the percentage of realization of registers in solving problem for both groups PC and MS. It shows that there is a considerable range in students' performance in converting semiotic registers from ASR to TVR (32%), and from TVR to NLR registers (35%), as well as a slight difference in the movement NLR to GR registers (7%), favoring the MS group.

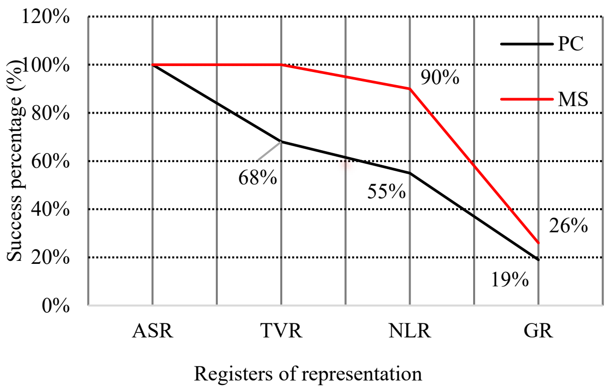

Figure 4. Comparison between PC and MS groups of successful conversion rates between registers of representation in numerical functions

5.2. Pupils' Performance in the Precision of the Curve of a Primitive Function

For the exercise related to determining the curve of a primitive function, we observed that 20 pupils (65%) of the sample of 31 pupils in the MS group did not respond. The percentage of pupils in the reduced MS group (11 pupils) who conducted the exercise successfully for the semiotic registers for the precision of a primitive function curve, is shown in Figure 5. The obtained results indicate that the average score of the reduced MS group is 16.77 with a lower value of 8 and higher of 20 and with a standard deviation of 3.23.

A first view of Figure 5 shows that all respondents haven't successfully completed the TVR representation register for this function, whereas the NLR and ASR registers have been successfully completed by 100% and 91% of learners, respectively. We notice that there is some difficulty in the conversion to ASR registers and a lot of difficulties in conversion to TVR one. The TVR registers difficulties exist because all the pupils described the variation table instead of drawing it (expressing the monotonicity of the function with sentences).

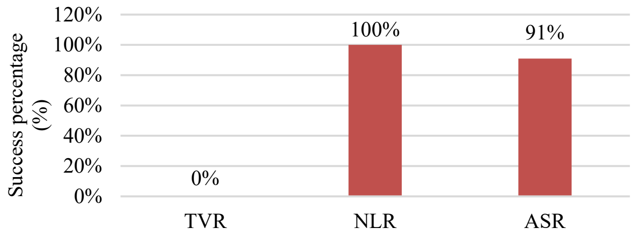

Figure 5. Percentage of realizing reduced MS group pupil's semiotic representation registers in the precision of the curve of a primitive function

6. DISCUSSION

The principal goal of the present study is to measure the capacity of pupils in solving numerical function problem and realizing semiotic representation registers related to this concept in order to compare the results of this objective between PC and MS Moroccan groups of the second-year baccalaureate (age 18). The overall results for both groups (62 pupils) were slightly higher than the average level, with an average score of 11.20 with a standard deviation of 3.67, revealing that the sample had difficulty in solving problems involving the processing of a complex numerical function. These findings differ from those obtained in the Australian context, where Australian students' mastery of functions in year 12 (age 18) may have played a significant impact in their algebraic generalization success rate of 70% over time [23]. The results of the problem test indicate that all of pupils passed the domain of definition successfully contrary to the difficulties existed in treatment of function limit in the vicinity of $+\infty$, treatment of variation table, treatment of the limit of $h(x)/x$ in the vicinity of $+\infty$, treatment of graphical function representation (drawing the treated function, the restriction function, the reciprocal function of the restriction function). The same difficulties were confirmed, by Hitt [24] where teachers and students of secondary education encounter difficulties of passage between representations of the concept of function, and by Sierpinska [25] where students have difficulty relating representations of functions. The analysis carried out in this problem reports that the results of MS group are better than PC group where MS group (M_{MS}=12.33> M_{PC}=10.08).

The differences in average scores among the PC and MS groups can be explained by the fact that the mathematical sciences stream is strictly reserved for pupils who have expressed an interest in this pathway and obtained good marks in mathematics and physics in the science common cycle, as well as by the difference in the number of hours of mathematics teaching allocated per week (7 hours for the MS group and 5 hours for the PC group). The physics-chemistry stream, on the other hand, is available to all scientific common cycle pupils who have fully shown an interest in following it. The Moroccan Ministry of Education organizes students' orientation and choice of academic path with the help of a guidance counselor and the official school management system "MASSAR", making it more credible. As indicated by Qetrani and Achtaich [26] and Ourahay [27], learners' scores reflect their procedural knowledge performance and those who score high do not have a deep conceptual understanding of functions. The same problem provides an overview of the pupil's ability to convert between four registers, i.e., 3 conversions, presented in Figure 6.

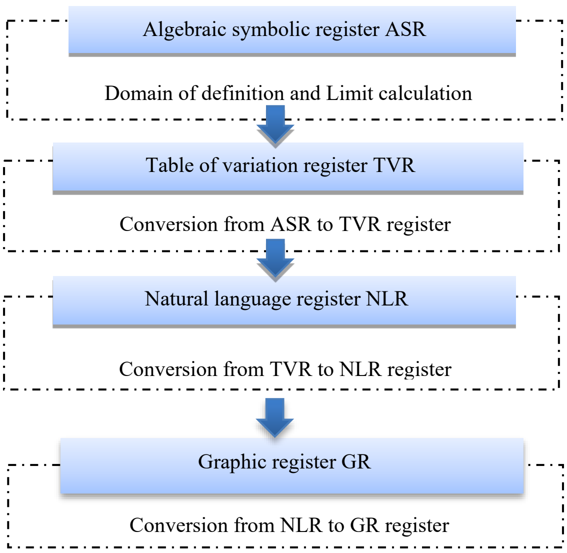

Figure 6. Registers of semiotic representations tested in the problem

All of the sample was relatively successful on the ASR, TVR and NLR registers because the TVR and NLR registers were answered correctly by just 84% and 73% pupils, respectively, even if they are others who have difficulties in treatment of variation table and limit of $h(x)/x$ in the vicinity of $+\infty$. But there is a great difficulty in realizing GR register because only 23% pupils who have succeeded and the rest didn't make it, knowing that the realization of registers doesn't imply the complete treatment of questions problem. Sajka [28] reported that students encounter difficulties in understanding function at different ages of high school education. The difficulties encountered in the realization of the questions associated with these registers indicate that the students have mental knowledge difficulties in the internal transformation of these registers.

According to the comparison between MS and PC groups in this meaning, the MS group has the best result in the performance of conversion semiotic registers: 32% of difference conversion from ASR to TVR register, 35% of difference conversion from TVR to NLR register, 7% of difference conversion from NLR to GR register. This difficulty of conversions is confirmed by Bloch [29] where high school students encounter barriers to learning numerical functions related to knowledge of the characteristics of the semiotic representation registers and the transition between them. The PC students have greater difficulties than MS students in the procedural knowledge of numerical function study. Previous studies have reported that high school students have difficulty understanding and solving mathematical problems involving functions as well as their representations in the semiotic representation registers [30, 22]. In addition, Qetrani and Achtaich [26] stated that Moroccan second–year baccalaureate students' grades are strongly correlated with the score for functions procedural knowledge (PK) of and much less correlated with the score for functions conceptual knowledge (CK). The difficulties of PK are responsible for the reduced means score. This has been confirmed by numerous studies in science didactics, which have shown that failure at school is due, in part, to learning difficulties [5].

The exercise studied in the second part is taught only for the MS group in the Moroccan mathematics curriculum. We aim to identify the difficulties of this group in the precision with a justification of a curve that represents the primitive of a derivable function (the function curve is indicated in the data) from proposals of graphical representation (each representation contains the tangent at the point of specific abscissa). The ability of this group to draw the table of variation of the primitive of the studied function and to use the directing coefficient of the tangent in the treatment is measured, respecting the conversion between registers mentioned in Figure 7.

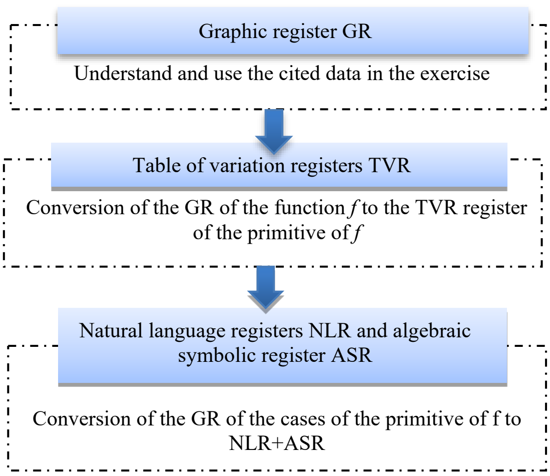

Figure 7. Registers of semiotic representations tested in the second exercise

The fact that 65% of copies were blank confirms that the learners in the sample had great difficulty in completing the exercise. Analysis of this 35% (reduced MS group) of papers indicates that all respondents did not do the TVR register and that pupils with a score of 20 expressed monotony with the NLR. This suggests that the pupils had difficulties moving from the GR register (represented by the graph in the statement) to the TVR register. Because the intermediate registers are not required, the key thing for them was to answer the question correctly by following their processing procedure in an unguided open question, rather than the details of the procedure leading to the correct response. Moreover, they did not all manage to convert the GR (the primitive graphs proposed in the statement) to the NLR and ASR registers to prove the understanding of their mental knowledge. Those difficulties are consistent. Similar results were obtained by Sierpinska [25], who stated that students have difficulties interpreting function graphs, and by Bloch [29], who stated that learners fail to correctly understand the nature of mathematical objects such as functions and limits from a graph of a function while asking them to deduce certain properties of that function. Thus, they have difficulties with procedural knowledge and understanding of mental and treatment activities.

7. CONCLUSION

Difficulties in solving a complex numerical function problem were explored and compared between PC and MS pupils in the Moroccan baccalaureate's second year. These difficulties are analyzed in terms of objectives that pupils should master to succeed in all the questions of the problem involving the concepts of domain of definition, limit, derivation, table of variation, and graphical representation, and the realization of semiotic registers associated to the various stages of a function's study. According to the results of this study, when learners solve a problem with a complex numerical function, they employ the quasi-automatic methods they learned in earlier issues, adapting them to the current circumstance. However, they meet into issues with processing (derivation, limits, graphical representation, etc.) and realizing semiotic representations of this function at various points.

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BIOGRAPHIES

Name: **Douaa**
Surname: **Bouanfir**
Birthday: 11.12.1992
Birthplace: Casablanca, Morocco
Bachelor: Science and Technology in Mathematics and Computer Science, Department of Mathematics, Faculty of Technical Sciences, Hassan II University, Casablanca, Morocco, 2016
Master: Teaching and Training Professions in Mathematics, Department of Mathematics, Science Faculty, Ibn Tofail University, Kenitra, Morocco, 2019
Doctorate: Student, Didactics, Educational Sciences, Teaching and Training Professions in Mathematics and Experimental Sciences, Department of Mathematics, Science Faculty, Ibn Tofail University, Kenitra, Morocco, Since 2019
The Last Scientific Position: Intern in Mathematics in High Schools, Provincial Direction of Ministry of National Education Preschool and Sport, Sidi Bernoussi Casablanca, Morocco, 2022-2023
Research Interests: Teaching Difficulties and Digital Function Learning

Name: **Mohammed**
Surname: **Abid**
Birthday: 07.10.1969
Birthplace: Sidi Bennour, Morocco
Bachelor: Chemistry, Department of Chemistry, Faculty of Sciences, IbnTofail University, Kenitra, Morocco, 1995
Master: Chemistry, Department of Chemistry, Faculty of Sciences, IbnTofail University, Kenitra, Morocco, 1997
Doctorate: Applied Chemistry, Department of Chemistry, Faculty of Sciences, IbnTofail University, Kenitra, Morocco, 2000
The Last Scientific Position: Prof., Regional Center for Education and Training Professions, Rabat, Morocco, sine 2011 and Associate Prof., Faculty of Sciences, Ibn Tofail University, Kenitra, Morocco, Since 2015
Research Interests: Science Didactic, Chemistry of glasses
Scientific Publications: 27 Papers, 1 Thesis

Name: **Bouazza**
Surname: **El Wahbi**
Birthday: 01.01.1967
Birthplace: Bouznika, Morocco
Bachelor: Fundamental Mathematics, Department of Mathematics, Faculty of Sciences, Mohamed V University, Rabat, Morocco, 1989
Master: Analysis Mathematics, Department of mathematics, Faculty of Sciences, Mohamed V University, Rabat, Morocco, 1993
Doctorate: Applied Mathematics, Department of Mathematic, Mohamed V University, Rabat, Morocco, 1998
The Last Scientific Position: Prof., Department of Mathematics, Science Faculty, Ibn Tofail University, Kenitra, Morocco, Since 2001
Research Interests: Mathematics and Analysis Application, Didactics of Mathematics Sciences, Analysis and Geometric Research and its Applications
Scientific Publications: 22 Papers, 2 Theses